

KANAZAWA-01-04

May, 2001

A light sterile neutrino based on the seesaw mechanism

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Abstract

We propose a simple model of the neutrino mass matrix which can explain the solar and atmospheric neutrino problems in a $3(\nu_L + \nu_R)$ framework. Assuming that only two right-handed neutrinos are heavy and a Dirac mass matrix has a special texture, we construct a model with four light neutrinos. The favorable structure of flavor mixings and mass eigenvalues required by those neutrino deficits is realized as a result of the seesaw mechanism. Bi-maximal mixing structure might be obtainable in this scheme. Since it contains a light sterile neutrino, it has a chance to explain the LSND result successfully. We consider an embedding of this scenario for the neutrino mass matrix into the SU(5) grand unification scheme using the Froggatt-Nielsen mechanism based on $U(1)_{F_1} \times U(1)_{F_2}$. Both a small mixing angle MSW solution and a large mixing angle MSW solution are obtained for the solar neutrino problem depending on the charged lepton mass matrix.

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1. Introduction

Recently the existence of non-trivial lepton mixing has been strongly suggested through the atmospheric and solar neutrino observations whose results can be explained by assuming the neutrino oscillations [1, 2, 3]. The predicted flavor mixing is much bigger than the one of quark sector. The explanation of this feature is a challenging issue for the construction of a satisfactory grand unified theory (GUT) and a lot of works have been done [4, 5]. In most of them the smallness of the neutrino mass is explained by the celebrated seesaw mechanism [6] and the flavor mixing structure is considered to be controlled by the Froggatt-Nielsen mechanism [7]. There are many works in which the Abelian flavor symmetry is discussed [8]. On the other hand, there is another experimental suggestion on the neutrino oscillation by the Liquid Scintillator Neutrino Detector (LSND) [9]. If we impose the simultaneous explanation of the result together with the atmospheric and solar neutrino deficits, it has been well-known that three different values of the squared mass difference are necessary. Then four light neutrinos including a sterile neutrino (ν_s) are required [10]. Various models of the sterile neutrino can be found in refs. [10-14]. Following the recent Super-Kamiokande analysis of the solar neutrino, the explanation of the solar neutrino problem based on the ν_e - ν_s oscillation seems to be disfavored [3]. It suggests that the (3+1)-neutrino spectrum might be a more favored scenario for the neutrino mass hierarchy than the (2+2)-scheme [15].

In this paper we consider a neutrino mass matrix in a $3(\nu_L + \nu_R)$ framework by using the seesaw mechanism. However, being different from the ordinary seesaw models our model contains a light right-handed neutrino as a result of the special texture of a right-handed Majorana neutrino mass matrix. Although there are the similar works in this direction, in most of them it is necessary to introduce the Majorana masses for the left-handed neutrinos in order to obtain simultaneously the required values of the mass eigenvalues and the flavor mixing angles as it can be found, for example, in [13, 14]. It means that an introduction of a new triplet Higgs field might be necessary. In the present model we only need the Dirac neutrino masses and the right-handed Majorana neutrino masses if we assume a special but simple texture in both of them at tree level. The model seems to have less parameters as compared to the previous ones.

One of the interesting points of the model is that the large mixing angle MSW solution for the solar neutrino problem can be consistently accommodated in the same way as other

solutions [16, 17]. The LSND result might be also explained if we take an appropriate solution for the solar neutrino problem [17]. Moreover, it is interesting that this scenario for the neutrino mass matrix could also be embedded into the GUT scheme by introducing a suitable flavor symmetry. Such an example in the SU(5) model will be constructed by fixing the charge assignment of quarks and leptons for that symmetry.

The organization of this paper is as follows. In section 2 we define our model and discuss its various phenomenological features in the case that the charged lepton mass matrix is diagonal. In section 3 we consider the embedding of the scenario into the SU(5) GUT scheme. We discuss the realization of the required form of the mass matrix in the basis of the Froggatt-Nielsen mechanism. The flavor structure in the quark sector is also discussed here. Section 4 is devoted to the summary.

2. A model of neutrino mass matrix

We consider a model defined by the following neutrino mass terms which are different from the usual seesaw model in the $3(\nu_L + \nu_R)$ framework:

$$-\mathcal{L}_{\text{mass}} = \sum_{\alpha} \sum_{p=2,3} m_{p\alpha} N_p \nu_{\alpha} + \sum_{p=2,3} m_{p1} N_p N_1 + \frac{1}{2} \sum_{p=2,3} M_p N_p N_p + \text{h.c.}, \quad (1)$$

where ν_{α} is an active neutrino ($\alpha = e, \mu, \tau$) and N_P ($P = 1 \sim 3$) is a charge conjugated state of the right-handed neutrino. We make the following assumption for the mass parameters in eq. (1):

$$m_{2e} = m_{2\mu} = m_{2\tau} \equiv \hat{\eta}, \quad m_{3e} \equiv \bar{\eta}_1, \quad m_{3\mu} = m_{3\tau} \equiv \bar{\eta}_2, \\ \hat{\eta} \sim \bar{\eta}_1 \sim \bar{\eta}_2 < m_{21} \sim m_{31} \ll M_2 \sim M_3, \quad (2)$$

where the mass parameters should be understood as their absolute values, although it is not expressed explicitly. A crucial difference from the usual seesaw model is that one of the right-handed neutrinos is assumed to be very light and also has very small mixings with other heavy right-handed neutrinos. We assume $M_{23} = 0$ in the Majorana mass matrix of N_P here, for simplicity. Following arguments are not largely changed even if we introduce the non-zero M_{23} . Under this assumption we can integrate out heavy right-handed neutrinos N_p and get the following 4×4 matrix as a result of the seesaw

mechanism¹,

$$m_\nu = \begin{pmatrix} A & B & B & D \\ B & C & C & E \\ B & C & C & E \\ D & E & E & F \end{pmatrix}. \quad (3)$$

The matrix elements $A \sim F$ are expressed by the model parameters in (2) as

$$\begin{aligned} A &= \frac{\hat{\eta}^2}{M_2} + \frac{\bar{\eta}_1^2}{M_3}, & B &= \frac{\hat{\eta}^2}{M_2} + \frac{\bar{\eta}_1 \bar{\eta}_2}{M_3}, & C &= \frac{\hat{\eta}^2}{M_2} + \frac{\bar{\eta}_2^2}{M_3}, \\ D &= \frac{\hat{\eta} m_{21}}{M_2} + \frac{\bar{\eta}_1 m_{31}}{M_3}, & E &= \frac{\hat{\eta} m_{21}}{M_2} + \frac{\bar{\eta}_2 m_{31}}{M_3}, & F &= \frac{m_{21}^2}{M_2} + \frac{m_{31}^2}{M_3}. \end{aligned} \quad (4)$$

If we define the diagonalization matrix U of the matrix (3) as $m_\nu^{\text{diag}} = U^T m_\nu U$, we find that U can be written as

$$U = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & -\sin \theta \sin \delta + \cos \theta \sin \gamma \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} (\cos \theta \sin \delta + \sin \theta \sin \gamma) \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} (\cos \theta \sin \delta + \sin \theta \sin \gamma) \\ -\sin \gamma & -\sin \delta & 0 & 1 \end{pmatrix}, \quad (5)$$

where $|\sin \gamma|, |\sin \delta| \ll 1$ is assumed and mixing angles are defined by

$$\tan 2\theta = \frac{2\sqrt{2}B}{A - 2C}, \quad \sin \gamma \simeq \frac{D \cos \theta + \sqrt{2}E \sin \theta}{F}, \quad \sin \delta \simeq \frac{-D \sin \theta + \sqrt{2}E \cos \theta}{F}. \quad (6)$$

The mass eigenvalues of m_ν are expressed as

$$\begin{aligned} m_1 &\simeq A \cos^2 \theta + \sqrt{2}B \sin 2\theta + 2C \sin^2 \theta, \\ m_2 &\simeq A \sin^2 \theta - \sqrt{2}B \sin 2\theta + 2C \cos^2 \theta, \\ m_3 &= 0, \quad m_4 = F, \end{aligned} \quad (7)$$

where we neglect the contribution from the fourth row and column of m_ν to $m_{1,2}$ taking account of the fact such as $A \gtrsim \frac{D^2}{F}$, $B \gtrsim \frac{DE}{F}$ and $C \gtrsim \frac{E^2}{F}$. Here we should note that in

¹It should be noted that the number of light sterile neutrinos is restricted at most to one in the present scenario. We obtain a 3×3 matrix if all of N_P are heavy. Even in such a case as far as we assume a proportional relation between $(m_{1\alpha})$ and $(m_{2\alpha})$ as vectors whose components are labeled by α , the texture for the active neutrinos is the same as eq. (3). Then it can be applied to the explanation of the solar and atmospheric neutrino problems in the same way as the following discussion. It is essentially the same as the one discussed in ref. [18], although it is derived in the different context.

(α, β)	(i, j)	$-4U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}(\equiv \mathcal{A})$	
(I, II)	$(1, 2)$	$\frac{1}{2} \sin^2 2\theta$	(A)
(I, III)	$(1, 2)$	$\frac{1}{2} \sin^2 2\theta$	(B)
(II, III)	$(1, 3)$	$\sin^2 \theta$	(C)
	$(2, 3)$	$\cos^2 \theta$	(D)
	$(1, 2)$	$-\frac{1}{4} \sin^2 2\theta$	(E)

Table 1. The contributions to each neutrino transition process $\nu_\alpha \rightarrow \nu_\beta$ from each sector (i, j) of the mass eigenstates.

this model the violation of the proportional relation between $(m_{2\alpha})$ and $(m_{3\alpha})$ as vectors is crucial to restrict a number of zero mass eigenvalue into one and to control the mixing structure. There is a freedom in a choice of two elements of $(m_{3\alpha})$ which are taken to be equal in (2). As far as we consider the case in which the charged lepton mass matrix is diagonal, it is not important. But when we consider the different situation, it might become crucial to the consideration of the oscillation phenomena. If the charged lepton mass matrix is diagonal, the above mixing matrix U is just the flavor mixing matrix $V^{(\text{MNS})}$ which controls the neutrino oscillation. We assume it in the charged lepton sector and also no CP violation in the lepton sector. At this stage we cannot determine to which flavor each ν_α corresponds so that we will use the Roman numerals for the subscript α for a while. Next we study the features of the oscillation phenomena in the model in order to fix the neutrino flavor.

The transition probability due to the neutrino oscillation $\nu_\alpha \rightarrow \nu_\beta$ after the flight length L is well-known to be written by using the matrix elements of (5) as

$$\mathcal{P}_{\nu_\alpha \rightarrow \nu_\beta}(L) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left(\frac{\Delta m_{ij}^2}{4E} L \right), \quad (8)$$

where $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$ and the weak eigenstate ν_α is related to the mass eigenstate $\tilde{\nu}_i$ by $\nu_\alpha = U_{\alpha i} \tilde{\nu}_i$ in the basis that the charged lepton mass matrix is diagonal. In Table 1 we summarize the contribution to each neutrino transition mode (α, β) from a sector (i, j) of the mass eigenstates. As a phenomenologically interesting case, we consider the situation that the mass eigenstates $\tilde{\nu}_1$ and $\tilde{\nu}_2$ are almost degenerate and the hierarchy $(m_3 \ll m_1 \sim m_2) \ll m_4$ among the mass eigenvalues is satisfied. This corresponds to a

well-known reversed hierarchy scenario for the atmospheric and solar neutrino problems in the (3+1)-neutrino spectrum [19]. The absolute value of each mass eigenvalue is smaller than the ordinarily discussed scenario because of $m_3 = 0$. Then every neutrino cannot be a hot dark matter candidate. If we apply it to explain the atmospheric and solar neutrino data, the squared mass difference should be taken as [1, 2]

$$2 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m_{13}^2 \simeq \Delta m_{23}^2 \lesssim 6 \times 10^{-3} \text{ eV}^2, \quad (9)$$

$$10^{-10} \text{ eV}^2 \lesssim \Delta m_{12}^2 \lesssim 1.5 \times 10^{-4} \text{ eV}^2. \quad (10)$$

A suitable value of Δm_{12}^2 should be chosen within the above range depending on which solution is adopted for the solar neutrino problem.

By inspecting Table 1 we find that the simultaneous explanation of both deficits of the atmospheric neutrino and the solar neutrino is possible if we identify the weak eigenstates of neutrinos (e, μ, τ) with (I, II, III). Under this identification the 3×3 submatrix of (5) is recognized as the correctly arranged MNS mixing matrix. If we note that $m_3 = 0$ and $\Delta m_{13}^2 \simeq \Delta m_{23}^2$ are satisfied, we find that the atmospheric neutrino is explained by $\nu_\mu \rightarrow \nu_\tau$ obtained as the combination of (C) and (D) in Table 1. This explanation is independent of the value of $\sin \theta$. On the other hand, the solar neutrino is expected to be explained by $\nu_e \rightarrow \nu_\mu$ (A) and also $\nu_e \rightarrow \nu_\tau$ (B). In both processes the amplitude $\mathcal{A} (\equiv -4 \sum U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j})$ is $\frac{1}{2} \sin^2 2\theta$. Thus if $\sin^2 2\theta \sim 10^{-2}$, the small mixing angle MSW solution (SMA) is realized [17]. In the case of $\sin^2 2\theta \sim 1$, it can give the large mixing angle MSW solution (LMA), the low mass MSW solution (LOW) and the vacuum oscillation solution (VO) [17] depending on the value of Δm_{12}^2 . The CHOOZ experiment [20] constrains a component U_{e3} of the MNS mixing matrix [21]. It comes from the fact that the amplitude \mathcal{A} of the contribution to $\nu_e \rightarrow \nu_x$ with the squared mass differences Δm_{13}^2 or Δm_{23}^2 always contains U_{e3} . The model is free from this constraint since $U_{e3} = 0$ is satisfied independently of the value of $\sin \theta$.

In order to see the viability of the scenario in a more quantitative way it is useful to estimate numerically what kind of tuning of the primary parameters in (1) and (2) is required to realize the suitable value for the oscillation parameters. For the convenience we introduce the following parametrization for the three light states:

$$\frac{\hat{\eta}}{\sqrt{M_2}} \equiv \mu^{\frac{1}{2}}, \quad \frac{\bar{\eta}_1}{\sqrt{M_3}} \equiv \epsilon_1 \mu^{\frac{1}{2}}, \quad \frac{\bar{\eta}_2}{\sqrt{M_3}} \equiv \epsilon_2 \mu^{\frac{1}{2}}. \quad (11)$$